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A Discussion of Disutility of Waiting

In this paper, we have considered a linear disutility of waiting, i.e., the disutility increases linearly in t, the expected time to have an order filled. The idea that customers are delay-sensitive and that the disutility of waiting is linear has been expressed by several other authors. For example, Van Ackere and Ninios (1993), Hassin (1986), Li (1992), Li and Lee (1994) and Stidham (1970) model the disutility of waiting in a similar, but more limited, manner compared to our model. Specifically, they assume that all customers have the same unit waiting cost, b, per unit time (i.e., customers are homogeneous and the disutility is linear). In Lederer and Li (1997), the disutility of waiting is heterogeneous (as in our model). However, a linear form is still assumed.

While most of the existing literature has assumed a linear form, it is quite possible that, for some consumers, the disutility would follow a more complex form. For example, one can imagine situations in which the disutility would be increasing and convex, reflecting the fact that consumers will tolerate small waits, but become increasingly distressed by long waits. Unfortunately, we have found that for even quite simple non-linear convex disutility functions, our analytical model becomes intractable. Here we briefly demonstrate the main difficulty in the analysis. Let $g(t)$ be a general function of t (the expected waiting time) so that the disutility of waiting for a customer with waiting sensitivity b is $b(g(t)+\alpha)$. Following the procedures described in the paper, we have

$$\theta_h = Pr\left\{b < \frac{p_l - p_h}{g(t)+\alpha}\right\} = \max\left[0, \min\left[1, \frac{p_l - p_h}{g(t)+\alpha}\right]\right].$$

To analyze CASE 3 (low-cost firm with in-stock policy vs. high-cost firm with stockless policy), we follow the procedures described in the proof of Proposition 2. Specifically, it is easy to see that the condition $\frac{\partial \pi_h^S}{\partial p_h} = 0$ is independent of T_h. Thus, the optimal p_h can be set independently of T_h. Solving $\frac{\partial \pi_h^S}{\partial p_h} = 0$, we obtain $p_h^*(p_l) = \frac{1}{2}(p_l + c_h)$.

We next consider the first order conditions for T_h. Using the fact that $p_h^*(p_l) = \frac{1}{2}(p_l + c_h)$ for any T_h, we have

$$\frac{\partial \pi_h^S}{\partial T_h} = -\frac{(p_l - c_h)^2 d}{4} \frac{g\prime\left(\frac{T_h}{2}\right)}{\left(g\left(\frac{T_h}{2}\right)+\alpha\right)^2} + \frac{A}{T_h^2} = 0$$

Although this equation yields closed-form solutions when $g\left(\frac{T_h}{2}\right)$ is a linear function, the solutions are intractable when $g\left(\frac{T_h}{2}\right)$ is a convex function. For example, when $g\left(\frac{T_h}{2}\right) = \left(\frac{T_h}{2}\right)^2$, the equation becomes a four degree polynomial which has no easily solvable roots.

Alternatively, if we take $g\left(\frac{T_h}{2}\right) = e^{\gamma\left(\frac{T_h}{2}\right)}$, then the above equation is not possible to solve analytically.

While we are unable to perform a complete analysis when the disutility of waiting is increasing and convex, we can comment on how the results of our model would likely change in this case. When a consumer's disutility increases more rapidly, as with an increasing and convex disutility function, we speculate that i) the price charged by the in-stock firm would increase, ii) the price charged by the stockless firm would increase, iii) the difference between the two prices would increase, i.e., the degree of stock-out compensation will increase, and iv) equilibria in which the two firms split the market will become less common. The intuition behind the first three points is based on the fact that, under a rapidly increasing and convex disutility function, customers quickly become very sensitive to the waiting time. Hence, the in-stock firm can increase its price and the stockless firm will likely also increase its price (to compensate for the expected decrease in its market share that would occur due to customers being more time-sensitive). However, to compensate for the customer's higher disutility, the stockless firm should increase the amount of stock-out compensation. We have verified this intuition through a small set of informal numerical tests. The intuition behind the fourth point is that competing with a stockless policy becomes more difficult as consumers become more sensitive to waiting time. Thus, for example, with a rapidly increasing disutility function we would expect the stockless firm to have more difficulty in capturing a significant portion of the market.

Finally, we note that identical results were observed when customers have a higher degree of fixed disutility (α). Therefore, we cautiously speculate that the impact of a rapidly increasing customer disutility would be similar to that of a higher degree of fixed disutility.

To summarize, while the main findings of this paper are derived assuming a linear disutility of waiting, we believe that our main result, i.e., that using a stockless operation with stock-out compensation can be a profitable alternative in a competitive market, will still hold.

B Technical Details & Proofs

Proof of Proposition 1. Let $\pi_i^{I,1}$ and $\pi_i^{I,2}$ be firm i's profit with market domination and market sharing. First, note that

$$\begin{aligned}
\pi_l^{I,1}(p_i) - \pi_h^{I,1}(p_i) &= (c_h - c_l)d + \sqrt{2A \cdot \gamma \cdot d} \cdot (\sqrt{c_h} - \sqrt{c_l}) > 0, \\
\pi_l^{I,2}(p_i) - \pi_h^{I,2}(p_i) &= (c_h - c_l)\frac{d}{2} + \sqrt{A \cdot \gamma \cdot d} \cdot (\sqrt{c_h} - \sqrt{c_l}) > 0,
\end{aligned}$$

and that all four of these profit functions are strictly increasing-linear functions of p_i. Therefore, the rest of the proof follows standard proof procedures of Bertrand competition (see, e.g., Tirole (1988)). Here we demonstrate why the equilibrium price does not cause any deviation. Note that at the equilibrium price, firm h's profit is zero with market domination. Since this price is also acceptable to firm l, firm l serves the market. Now, firm h needs to exit the market because it will have negative profit if it shares the market at the equilibrium price. ■

Proof of Lemma 1. Let $NS_i(b_k)$ be net surplus of a customer with waiting sensitivity b_k from purchasing a product from firm i (i.e., $v - p_i - \left(\frac{T_i}{2} + \alpha\right) b_k$). First, note that $NS_i(b_k)$ is a linear-decreasing function of b_k. Therefore, combining the precondition ($\{p_l > p_h, T_l < T_h\}$ or $\{p_l < p_h, T_l > T_h\}$) and the linearity implies that there exists at most one intersection (i.e., $\tilde{b_s}$) on b_k. Next, the precondition of strictly positive market share implies $\tilde{b_s} < 1$, which implies that $NS_l(b_k)$ and $NS_h(b_k)$ intersect where $b_k < 1$.

Now consider the case where $p_l > p_h$ and $T_l < T_h$. Under this case, we see that $NS_h(b_k) > NS_l(b_k)$ where $b_k < \tilde{b_s}$ and $NS_h(b_k) < NS_l(b_k)$ where $b_k > \tilde{b_s}$. Therefore, it is easy to see that 1) $\tilde{b_s} < \tilde{b_h} < \tilde{b_l}$, 2) customers whose waiting sensitivity is less than $\tilde{b_s}$ prefer firm h, and 3) the other customers whose waiting sensitivity is greater than $\tilde{b_s}$ AND net surplus is greater than 0 prefer firm l. The proof of the other case $\{p_l < p_h,\, T_l > T_h\}$ is exactly identical to this procedure. ■

Proof of Proposition 2. ***Part i) & ii):*** We start by considering the first order conditions for p_h. We first note that the condition $\frac{\partial \pi_h^S}{\partial p_h} = 0$ is independent of T_h. Thus, the optimal p_h can be set independently of T_h. Solving $\frac{\partial \pi_h^S}{\partial p_h} = 0$, we obtain $p_h^*(p_l) = \frac{1}{2}(p_l + c_h)$.

We next consider the first order conditions for T_h. Solving $\frac{\partial \pi_h^S}{\partial T_h} = 0$, and using the fact that $p_h^*(p_l) = \frac{1}{2}(p_l + c_h)$ for any T_h, we find two possible solutions for $T_h^*(p_l)$. However, one of these solutions is strictly negative and can be ruled out. Thus we are left with $T_h^*(p_l) = \frac{-4\alpha\sqrt{A}}{2\sqrt{A}-\sqrt{2d}\sqrt{(c_h-p_l)^2}}$. Next, it is easy to show that, in order to ensure $T_h^*(p_l) > 0$, we need either $p_l < c_h - \sqrt{\frac{2A}{d}}$ or $p_l > c_h + \sqrt{\frac{2A}{d}}$.

The next step in the proof is to check the second order conditions for the optimality of $p_h^*(p_l)$ and $T_h^*(p_l)$. As noted above, the optimal p_h can be set independently of T_h. Thus, we first verify the second order conditions for p_h. It is easy to show that, for any $T_h > 0$, $\frac{\partial^2 \pi_h^S}{\partial p_h^2} = \frac{-4d}{T_h + 2\alpha} \leq 0$. Thus, the second order conditions for p_h are satisfied under the condition on p_l assumed in the statement of the proposition.

Next, assuming that $p_h = p_h^*(p_l)$ for any T_h, we can verify the second order conditions for T_h. It is straightforward to show that under the conditions on p_l stated in the proposition, we have $\frac{\partial^2 \pi_h^S}{\partial T_h^2} \leq 0$ at the point $T_h^*(p_l)$. Thus, $T_h^*(p_l)$ is a local maximum. Next, we can show that $\frac{\partial \pi_h^S}{\partial p_h} < 0$ for $T_h > T_h^*(p_l)$ and $\frac{\partial \pi_h^S}{\partial p_h} > 0$ for $T_h < T_h^*(p_l)$. Thus, the profit function is always increasing for $T_h < T_h^*(p_l)$ and decreasing for $T_h > T_h^*(p_l)$, i.e., the profit function is unimodal, and thus $T_h^*(p_l)$ must be a global maximum, given that $p_h = p_h^*(p_l)$.

Part iii): We need to ensure that $0 < \theta_h^*(p_l) < 1$, i.e., that $p_h^*(p_l) < p_l < p_h^*(p_l) + \alpha + \frac{T_h^*(p_l)}{2}$. To ensure that $p_l > p_h^*(p_l) = \frac{1}{2}(p_l + c_h)$, we need $p_l > c_h$. To ensure that $p_l < p_h^*(p_l) + \alpha + \frac{T_h^*(p_l)}{2} = \frac{1}{2}(p_l + c_h) + \alpha + \frac{T_h^*(p_l)}{2}$, we need $p_l < c_h + T_h^*(p_l) + 2\alpha$. If we plug in $T_h^*(p_l)$, using the fact that $p_l > c_h$, we find that $p_l < p_h^*(p_l) + \alpha + \frac{T_h^*(p_l)}{2}$ will hold only if $p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$. Thus, we have shown that $0 < \theta_h^*(p_l) < 1$ will hold if $c_h < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$.

The conditions required for $T_h^*(p_l) > 0$ are $p_l < c_h - \sqrt{\frac{2A}{d}}$ or $p_l > c_h + \sqrt{\frac{2A}{d}}$, while the conditions required for $0 < \theta_h^*(p_l) < 1$ are $c_h < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$. Combining these two sets of conditions, we obtain $c_h + \sqrt{\frac{2A}{d}} < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$, as assumed in the statement of the proposition.

Part iv): We next show that, under the condition on p_l in the statement of the proposition, $\pi_h^{S*}(p_l) > 0$. We can write $\pi_h^{S*}(p_l) = (p_h^*(p_l) - c_h)d\ \theta_h^*(p_l) - \frac{A}{T_h^*(p_l)} = \frac{d}{2}\frac{(p_l-c_h)^2}{T_h^*(p_l)+2\alpha} - \frac{A}{T_h^*(p_l)}$, where the last step follows from the fact that $p_l - p_h^*(p_l) = \frac{1}{2}(p_l - c_h)$. The condition $\pi_h^{S*}(p_l) > 0$ can now be rewritten as $\frac{T_h^*(p_l)}{T_h^*(p_l)+2\alpha} > \frac{2A}{d(p_l-c_h)^2}$.

Plugging in for $T_h^*(p_l)$ and simplifying, we obtain $\frac{T_h^*(p_l)}{T_h^*(p_l)+2\alpha} = \sqrt{\frac{2A}{d}}\frac{1}{\sqrt{(c_h-p_l)^2}}$. Thus, the condition $\pi_h^{S*}(p_l) > 0$ requires that $\sqrt{\frac{2A}{d}}\frac{1}{\sqrt{(c_h-p_l)^2}} > \frac{2A}{d(p_l-c_h)^2}$. Simplifying and using the fact that $p_l > c_h$, we obtain $p_l > c_h + \sqrt{\frac{2A}{d}}$. Thus, under the condition on p_l in the proposition, $\pi_h^{S*}(p_l) > 0$.

Part v): We next show that $\pi_h^{S*}(p_l)$ is an increasing function of p_l. Taking the derivative with respect to p_l, we have:

$$\frac{\partial \pi_h^{S*}}{\partial p_l} = \frac{d(p_l - c_h)}{T_h^*(p_l) + 2\alpha} + \left(\frac{A}{T_h^{*2}(p_l)} - \frac{d}{2}\frac{(p_l - c_h)^2}{(T_h^*(p_l) + 2\alpha)^2}\right)\frac{dT_h^*(p_l)}{dp_l}.$$

Using the proof of ***Part iv)***, we can show that $\left(\frac{A}{T_h^{*2}(p_l)} - \frac{d}{2}\frac{(p_l-c_h)^2}{(T_h^*(p_l)+2\alpha)^2}\right) = 0$. Thus, $p_l > c_h$ implies that $\frac{\partial \pi_h^{S*}}{\partial p_l} > 0$. ■

Proof of Proposition 3. To prove this proposition, we proceed in two steps: i) we show that $\pi_l^I(p_l|p_h^*, T_h^*) = 0$ when $p_l = c_h + \sqrt{\frac{2A}{d}}$ and ii) we prove that $\frac{\partial \pi_l^I(p_l|p_h^*,T_h^*)}{\partial p_l} > 0$ if $c_h + \sqrt{\frac{2A}{d}} < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$. Together, these two results ensure that $\pi_l^I(p_l|p_h^*, T_h^*) > 0$ for p_l satisfying $c_h + \sqrt{\frac{2A}{d}} < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$.

To prove Part i), we first write $\pi_l^I(p_l|p_h^*, T_h^*) = (p_l - c_l)d(1 - \theta_h^*(p_l)) - \sqrt{2A\gamma c_l d(1 - \theta_h^*(p_l))}$, where $\theta_h^*(p_l) = \frac{2(p_l - p_h^*)}{T_h^* + 2\alpha}$ implies that $1 - \theta_h^*(p_l) = \frac{1}{2\alpha}\left(1 - \sqrt{\frac{2A}{d}}\frac{1}{p_l - c_h}\right)$. Next, note that $p_l = c_h + \sqrt{\frac{2A}{d}}$ implies that $\frac{1}{p_l - c_h} = \sqrt{\frac{d}{2A}}$. Thus, when $p_l = c_h + \sqrt{\frac{2A}{d}}$, we have $1 - \theta_h^*(p_l) = 0$. Therefore, when $p_l = c_h + \sqrt{\frac{2A}{d}}$, we have $\pi_l^I(p_l|p_h^*, T_h^*) = 0$.

Next, to prove Part ii), we plug $1 - \theta_h^*(p_l)$ into $\pi_l^I(p_l|p_h^*, T_h^*)$, take the derivative with respect to p_l, and simplify to get $\frac{\partial \pi_l^I(p_l|p_h^*, T_h^*)}{\partial p_l} = \frac{d}{2\alpha}\left(1 - \sqrt{\frac{2A}{d}}\frac{1}{p_l - c_h}\left(1 - \frac{p_l - c_l}{p_l - c_h}\right)\right)$. Finally, using the facts that $p_l > c_h$ and $c_h > c_l$, we know that $\frac{1}{p_l - c_h} > 0$ and $(1 - \frac{p_l - c_l}{p_l - c_h}) < 0$. Therefore, $\frac{\partial \pi_l^I(p_l|p_h^*, T_h^*)}{\partial p_l} > \frac{d}{2\alpha} > 0$, which completes the proof. ■

Proof of Proposition 4. We show that $\pi_l^I(p_l|p_h, T_h)$ is a unimodal function of p_l, where $\pi_l^I(p_l|p_h, T_h) = (p_l - c_l)d(1 - \theta_h(p_l)) - \sqrt{2A\gamma c_l d(1 - \theta_h(p_l))}$, where $\theta_h(p_l) = \frac{2(p_l - p_h)}{T_h + 2\alpha}$. We take the first derivative, set it equal to 0, and rewrite the first order condition as

$$d\left(1 - \frac{2(p_l - p_h)}{T_h + 2\alpha}\right) + (p_l - c_l) \cdot d\left(-\frac{2}{T_h + 2\alpha}\right) = -\frac{2A\gamma c_l d}{(T_h + 2\alpha)\sqrt{\left(1 - \frac{2(p_l - p_h)}{T_h + 2\alpha}\right)}}. \tag{9}$$

The LHS of Eq. (9) is a decreasing linear function of p_l, while the RHS is a decreasing concave function. We evaluate two functions at $p_l = 0$, and see that $LHS|_{p_l=0} > 0$ and $RHS|_{p_l=0} < 0$. Combining these two observations implies that Eq. (9) has at most two real solutions.

When there exists no solution or two identical solutions, it is easy to see that $\frac{\partial \pi_l^I(p_l|p_h, T_h)}{\partial p_l} \geq 0$, which implies that $\pi_l^I(p_l|p_h, T_h)$ is a strictly increasing function. Now, consider the case in which there are two different real solutions. We note that $\frac{\partial \pi_l^I(p_l|p_h, T_h)}{\partial p_l}|_{p_l=0} > 0$ and $\frac{\partial \pi_l^I(p_l|p_h, T_h)}{\partial p_l}|_{p_l=p_l^U} = -\frac{d(T_h + 2\alpha + 2(p_h - c_l))}{T_h + 2\alpha} < 0$, where $p_l^U = \left(p_h + \frac{T_h}{2} + \alpha\right)$ is an upper bound on p_l derived from $\frac{2(p_l - p_h)}{T_h + 2\alpha} \leq 1$. This implies that p_l^U is located between two stationary points. Therefore, $\pi_l^I(p_l|p_h, T_h)$ is a unimodal function of p_l in $[0, p_l^U]$. This completes the proof. ■

Proof of Proposition 5. We prove this result by contradiction. Suppose that there is an equilibrium that satisfies $p_l^E \geq p_h^E + \frac{T_h^E}{2} + \alpha$ (by implication, $\pi_l^I = 0$), $\pi_h^S > 0$, and the other conditions of Proposition 5. Let $p_l^{L,M}$ be the lower bound of firm l's price under a monopolistic setting with an in-stock policy (i.e., $\pi_l^{I,M}(p_l^{L,M}) = 0$).

First, consider $p_h^E > p_l^{L,M}$. For this case, we construct a pricing policy that generates positive profit for firm l. To this end, note that firm l can set $p_l = p_h^E$ and make positive profit, since all customers prefer firm l (i.e., $\tilde{b}_s = 0$ & $\pi_l^{I,M}(p_l^{L,M} + \epsilon) > 0$).

Second, consider the other case in which $p_h^E \leq p_l^{L,M}$, and suppose that there exists an equilibrium. We show that firm h is unable to generate positive profit. To this end, we solve

$$\begin{aligned} \max \quad & \pi_h^S(p_h, T_h) = (p_h - c_h) \cdot d \cdot \max\left[\frac{2(1-p_h)}{T_h + 2\alpha}, 1\right] - \frac{A}{T_h} \\ s.t. \quad & p_h \leq p_l^{L,M} = c_l + \sqrt{\frac{2A\gamma c_l}{d}}, \\ & p_h + \frac{T_h}{2} + \alpha \leq p_l^L = c_h + \sqrt{\frac{2A}{d}}. \end{aligned}$$

The second constraint is derived from the fact that if firm h sets $p_h^E + \frac{T_h^E}{2} + \alpha > p_l^L$, then firm l would deviate by setting $p_l = \left(p_h^E + \frac{T_h^E}{2} + \alpha\right) - \epsilon$ and have positive profit (refer to Propositions 2 and 3). First, note that the second constraint implies that $\max\left[\frac{2(1-p_h)}{T_h+2\alpha}, 1\right] = 1$. Hence, $\pi_h^S(p_h, T_h)$ becomes $(p_h - c_h) \cdot d - \frac{A}{T_h}$. In addition, we see that the second constraint is binding, i.e., $p_h = c_h + \sqrt{\frac{2A}{d}} - \left(\frac{T_h}{2} + \alpha\right)$. Now, we rewrite the problem as

$$\begin{aligned} \max \quad & \pi_h^S(p_h, T_h) = \left(\sqrt{\frac{2A}{d}} - \left(\frac{T_h}{2} + \alpha\right)\right) d - \frac{A}{T_h} \\ s.t. \quad & 2\left[c_h + \sqrt{\frac{2A}{d}} - \left(c_l + \sqrt{\frac{2A\gamma c_l}{d}}\right) - \alpha\right] p_h \leq T_h. \end{aligned}$$

Solving for the optimal solutions, the unconstrained problem has two stationary points, i.e., $T_i^* = \sqrt{\frac{2A}{d}}$ or $T_i^* = -\sqrt{\frac{2A}{d}}$. It is easy to see that $T_i^* = \sqrt{\frac{2A}{d}}$ is a global maximum. By plugging into the profit function and simplifying, we have $\pi_h^S(p_h, T_h) = -d \cdot \alpha$. Therefore, firm h cannot make positive profit in this range. This contradicts the equilibrium assumption and completes the proof. ■

Proof of Proposition 6. Note that the equilibrium should satisfy $p_l^E \leq p_h^E$ (by implication, $\pi_h^S = 0$ & $\pi_l^I > 0$). Next, let p_l^L and p_l^U be the two bounds of Proposition 2, i.e., $p_l^L = c_h + \sqrt{\frac{2A}{d}}$ and $p_l^U = c_h + \sqrt{\frac{2A}{d}} + 2\alpha$. Since $p_l^L < p_l^U$, we consider three scenarios:

Scenario 1: $p_l^U \leq p_l^E$: Note that there is no equilibrium that satisfies the preconditions of Proposition 6 in this region, because firm h can reduce p_h sufficiently so that it can dominate the market.

Scenario 2: $p_l^L < p_l^E < p_l^U$: There exists a unique solution so that two firms share the market (refer to Proof of Proposition 2), but note that the solution does not satisfy $p_l^E \leq p_h^E$.

Scenario 3: $p_l^E \leq p_l^L$: Suppose that firm l sets $p_l = p_l^L$. Then, firm h can consider two strategies: dominating or sharing the market. Since dominating the market is not feasible (see Proof of Proposition 5), we only need to consider sharing the market. We demonstrate that firm h cannot make positive profit with this strategy. To this end, we solve

$$\begin{aligned} \max \quad & \pi_h^S(p_h, T_h | p_l^L) = (p_h - c_h) \cdot d \cdot \frac{2(p_l^L - p_h)}{T_h + 2\alpha} - \frac{A}{T_h} \\ s.t. \quad & \frac{2(p_l^L - p_h)}{T_h + 2\alpha} \leq 1, \end{aligned}$$

where $p_l^L = c_h + \sqrt{\frac{2A}{d}}$. By taking the first derivative with respect to p_h and setting it equal to 0, we have $p_h^* = \frac{p_l^L + c_h}{2}$. Simplifying the profit function with p_h^* and p_l^L, we have $\pi_h^S(T_h | p_h^*, p_l^L) = -\frac{2A\alpha}{T_h(T_h+2\alpha)}$, which implies that firm h is unable to make positive profit. ■