Appendices

A. Derivation of $R(i,x)$

Define $V(i,x)$ as the retailer's optimal total expected discounted profit attainable starting with state (i,x). We find

$$V(i,x) = \max_{y \geq x} \left\{ \begin{array}{l} -w_i(y-x) + r_i E[\min(y,D_i)] - h_i E(y-D_i)^+ \\ +\beta(1-q_i)p_{i+1}E(y-D_i)^+ \\ +\beta \left[\begin{array}{l} q_i(s_{i+1}E(y-D_i)^+ + V(0,0)) \\ +(1-q_i)EV(i+1,(y-D_i)^+) \end{array} \right] \end{array} \right\}. \quad \text{(A.1)}$$

Inside the optimization operator, $w_i(y-x)$ is the procurement cost; $r_iE[\min(y,D_i)]$ is the expected one-period sales revenue; $h_iE(y-D_i)^+$ is the expected one-period holding cost. With probability q_i, the following period is an introduction period and the current cycle ends. The leftover inventory will be returned to the manufacturer at buyback price s_{i+1} per unit and the retailer's system re-starts in state (0,0). With probability $1-q_i$, the following period is not an introduction period and the age of the initial inventory at the beginning of the following period will be $i+1$. So the system enters state $(i+1,(y-D_i)^+)$ and the retailer receives a price protection fee of $p_{i+1}E(y-D_i)^+$.

Since $w_i x$ is a constant for given (i,x), we apply the classic technical trick developed in Veinott (1966) to reformulate (A.1). Let $R(i,x)=V(i,x)-w_i x$, where $R(i,x)$ is the maximal expected discounted profit assuming that we are charged for the initial inventory at the cost rate w_i. With some algebra, we can re-write the dynamic programming problem for the retailer with age $i\geq 0$ and initial inventory $x\geq 0$ as follows:

$$R(i,x)=\max_{y\geq x}\left\{g_R(i,y)+\beta[(1-q_i)ER(i+1,(y-D_i)^+)+q_iR(0,0)]\right\}$$

where

$$g_R(i,y)=(r_i-w_i)E[\min(y,D_i)]-[h_i+w_i-\beta q_i s_{i+1}-\beta(1-q_i)(w_{i+1}+p_{i+1})]E(y-D_i)^+.$$

Similar approach can be used to derive $T(i,x)$, $R^s(i,x,j)$ and $T^s(i,x,j)$. We directly give these expressions in the text and omit the intermediate procedures. ■

B. Optimality Equations with Age-independent Data

For the baseline model, the optimality equation is

$$T(x)=\max_{y\geq x}\left\{g(y)+\beta[(1-q)ET((y-D)^+)+qT(0)]\right\}, \tag{B.1}$$

where the myopic function is

$$g(y)=(r-c)E[\min(y,D)]-[h+c-\beta qb-\beta(1-q)c]E(y-D)^+. \tag{B.2}$$

The optimal solution for $T(x)$ is myopic, i.e, $y^*=F^{-1}(\frac{r-c}{r+h-\beta qb-\beta(1-q)c})$.

For the information-sharing model, when $j=1$, the optimality equation for the supply chain is

$$T^s(x,1)=\max_{y\geq x}\left\{\begin{array}{l}(r-c)E[\min(y,D)]-(c+h-\beta b)E(y-D)^+\\+q\beta R^s(0,1)+(1-q)\beta R^s(0,0)\end{array}\right\}, \tag{B.3}$$

when $j = 0$, the optimality equation is

$$T^s(x,0) = \max_{y \geq x} \left\{ \begin{array}{l} (r-c)E[\min(y,D)] - (c+h-\beta c)E(y-D)^+ \\ +\beta q ET^s((y-D)^+,1) + \beta(1-q)ET^s((y-D)^+,0) \end{array} \right\}. \tag{B.4}$$

The optimal solution for $T^s(x,1)$ is myopic but that for $T^s(x,0)$ is not myopic and needs to be solved by iteration. ■

C. Proof of Lemma 1

To facilitate the analysis, we truncate $R(i,x)$ to $R_{(n)}(i,x)$, where $R_{(n)}(i,x)$ is the retailer's maximal expected profit starting in state (i,x) when there are n periods remaining. When $n = 1$,

$$R(i,x) = \max_{y \geq x} \{g_R(i,y)\} = \max_{y \geq x} \left\{L_{(1)}(i,y)\right\}.$$

It is well-known that for any given i, $g_R(i,y)$ is a newsvendor function with profit margin, $r_i - w_i$, and overstock cost, o_i. Under Assumption 1, we see that $r_i - w_i$ and o_i are positive. Since the density function $f_i(\cdot) > 0$, it can be shown that $g_R(i,y)$ is strictly concave in y for any given i. Solving

$$\frac{\partial g_R(i,y)}{\partial y} = 0,$$

we have

$$y_i^{(1)} = F_i^{-1}\left(\frac{r_i - w_i}{r_i - w_i + o_i}\right).$$

The optimal inventory policy is to order-up-to $y_i^{(1)}$ if $y_i^{(1)} > x$; and no order is placed if $y_i^{(1)} \leq x$. We express $H_{(1)}(i,x)$ as the following.

$$R_{(1)}(i,x) = \left\{ \begin{array}{ll} L_{(1)}(i,y_i^{(1)}) & x < y_i^{(1)}, \\ L_{(1)}(i,x) & x \geq y_i^{(1)}. \end{array} \right.$$

Following the above equation, one can verify that for any given i, $R_{(1)}(i,x)$ is non-increasing and concave in x.

Next, we hypothesize that for some n, $R_{(n)}(i,x)$ is non-increasing and concave in x.

When there are $n+1$ periods remaining, the optimality equation is

$$\begin{aligned} R_{(n+1)}(i,x) &= \max_{y\geq x}\left\{g_R(i,y)+\beta[(1-q_i)ER(i+1,(y-D_i)^+)+q_iR(0,0)]\right\} \\ &= \max_{y\geq x}\left\{L_{(n+1)}(i,y)\right\}. \end{aligned}$$

We shall prove that $L_{(n+1)}(i,y)$ has two properties:

(i) $L_{(n+1)}(i,y)\to-\infty$ when $y\to\infty$;

(ii) $L_{(n+1)}(i,y)$ is strictly concave in y for any given i.

The first derivative of $g_R(i,y)$ with respect to y is

$$\frac{\partial g_R(i,y)}{\partial y}=(r_i-w_i)(1-F_i(y))-o_iF_i(y).$$

We see that when $y\to\infty$, $\frac{\partial g_R(i,y)}{\partial y}\to -o_i$ and $g_R(i,y)\to-\infty$. We also know that $R_{(n)}(i+1,(y-D_i)^+)$ is non-increasing in y. Hence, when $y\to\infty$, $L_{(n+1)}(i,y)\to-\infty$. This proves the first property of $L_{(n+1)}(i,y)$.

Note that $(y-d)^+$ is a convex and non-decreasing function of y for any given d. By the induction hypothesis, $R_{(n)}(i+1,x)$ is non-decreasing and concave in y. Using the fact that if $f(y)$ is non-increasing and concave in y and $g(y)$ is non-decreasing and convex in y, then $f(g(y))$ is non-increasing and concave in y. (Using the chain rule twice, one can show that the second derivative of $f(g(y))$ is non-positive). We conclude that for any realized $D_i=d$, $R_{(n)}(i+1,(y-d)^+)$ is non-increasing and concave in y. Since the expectation operator preserves the concavity and non-increasing property, $ER_{(n)}(i+1,(y-D_i)^+)$ is non-increasing and concave in y. This further implies that $L_{(n+1)}(i,y)$ is a strictly concave function of y (since $g_R(i,y)$ is strictly concave in y). This proves the second property of $L_{(n+1)}(i,y)$.

The two properties of $L_{(n+1)}(i,y)$ guarantee that there exist a unique solution (denoted by $y_i^{(n+1)}$) that maximizes $L_{(n+1)}(i,y)$. The optimal inventory policy is to order-up-to $y_i^{(n+1)}$ if $y_i^{(n+1)}>x$, and not to order if $y_i^{(n+1)}\leq x$. We express $R_{(n+1)}(i,x)$ as

$$R_{(n+1)}(i,x)=\begin{cases} L_{(n+1)}(i,y_i^{(n+1)}) & x<y_i^{(n+1)}, \\ L_{(n+1)}(i,x) & x\geq y_i^{(n+1)}. \end{cases}$$

Using the strict concavity of $R_{(n+1)}(i, y)$ in y, one can verify that for any given i, $R_{(n+1)}(i, x)$ is non-increasing and concave in x. This completes the proof of induction.

Finally, we let $n \to \infty$ and $L_{(n)}(i, y) \to L(i, y)$. We conclude that $L(i, y)$ is concave in y for any given i, thus, an age-dependent base-stock policy is optimal. ■

D. Proof of Lemma 3

Notice that if $\frac{c}{d} = \frac{e}{f} = \rho$, then

$$\rho = \frac{cX + eY}{dX + fY},$$

where c, d, e, f, X, Y are non-zero numbers. Recall that

$$g_R(i, y) = (r_i - w_i)E[\min(y, D_i)] - o_i E(y - D_i)^+$$

and that

$$g(i, y) = (r_i - c)E[\min(y, D_i)] - (h_i + c - k_i)E(y - D_i)^+.$$

We immediately see that when

$$\frac{r_i - w_i}{r_i - c} = \frac{o_i}{h_i + c - k_i} = \rho, \tag{D.1}$$

it holds that $g_R(i, y) = \rho g(i, y)$ for any i and y. ■

E. Proof of Proposition 1

To facilitate the analysis, we truncate $R(i, x)$ to $R_{(n)}(i, x)$, where $R_{(n)}(i, x)$ is the retailer's maximal expected profit starting in state (i, x) when there are n periods remaining. When $n = 1$, from the Proof of Lemma 1, we see that the optimal inventory policy is to order-up-to $z_i^{(1)}$ if $z_i^{(1)} > x$; and no order is placed otherwise, where $z_i^{(1)}$ is

$$z_i^{(1)} = F_i^{-1}\left(\frac{r_i - w_i}{r_i - w_i + o_i}\right).$$

Likewise, we truncate $T(i,x)$ to $T_{(n)}(i,x)$, where $T_{(n)}(i,x)$ is the supply chain maximal expected profit starting in state (i,x) when there are n periods remaining. When $n=1$,

$$T_{(1)}(i,x) = \max_{y \geq x} \{g(i,y)\}.$$

Solving the first order condition, we have

$$y_i^{(1)} = F_i^{-1}\left(\frac{r_i - c}{r_i + h_i - k_i}\right).$$

The optimal inventory policy is to order-up-to $y_i^{(1)}$ if $y_i^{(1)} > x$; and no order is placed otherwise. When equation (4.6) is true, it can be seen that

$$\frac{r_i - w_i^*}{r_i - w_i^* + o_i^*} = \frac{r_i - c}{r_i + h_i^* - k_i^*}.$$

So $y_i^{(1)} = z_i^{(1)}$ (i.e., the supply chain is coordinated). As a direct result of Lemma 3, we see that $R_{(1)}(i,x) = \rho T_{(1)}(i,x)$ for any i and x. Next, we hypothesize that for some n, it holds that $R_{(n)}(i,x) = \rho T_{(n)}(i,x)$ for any i and x. When there are $n+1$ periods remaining, the optimality equation for the retailer is

$$\begin{aligned} R_{(n+1)}(i,x) &= \max_{y \geq x} \left\{g_R(i,y) + \beta[(1-q_i)ER_{(n)}(i+1,(y-D_i)^+) + q_i R_{(n)}(0,0)]\right\} \quad \text{(E.1)} \\ &= \max_{y \geq x} \left\{\rho g(i,y) + \beta[(1-q_i)\rho ET_{(n)}(i+1,(y-D_i)^+) + \rho q_i T_{(n)}(0,0)]\right\} \\ &= \rho T_{(n+1)}(i,x). \end{aligned}$$

Finally, we let $n \to \infty$ and $T_{(n)}(i,x) \to T(i,x)$ and $R_{(n)}(i,x) \to R(i,x)$. We conclude that $R(i,x) = \rho T(i,x)$ for any i and x. ■

F. Proof of Corollary 1

Suppose that ρ_i is the profit sharing ratio when the age of the cycle is i. From equation (E.1), we see that

$$\begin{aligned} R_{(n+1)}(i,x) &= \max_{y \geq x} \left\{g_R(i,y) + \beta[(1-q_i)ER_{(n)}(i+1,(y-D_i)^+) + q_i R_{(n)}(0,0)]\right\} \\ &= \max_{y \geq x} \left\{\rho_i g(i,y) + \beta[(1-q_i)\rho_{i+1} ET_{(n)}(i+1,(y-D_i)^+) + \rho_0 q_i T_{(n)}(0,0)]\right\}. \end{aligned}$$

Because $\frac{\partial g(i,y)}{\partial y} \neq \frac{\partial}{\partial y} ET_{(n)}(i+1,(y-D_i)^+)$, we see that only when $\rho_0 = \rho_1 = ... = \rho$, the optimal solution for $R_{(n+1)}(i,x)$ equals to that for $T_{(n+1)}(i,x)$. ■

G. Proof of Proposition 3

First, suppose that the chain optimal policy for the baseline model is $\{y_i^*\}$. In the information-sharing model, for each pair of (i, j) there exists a corresponding policy that uses y_i^* as the base-stock level regardless of j. Such a policy is equivalent to ignoring the shared information and performs as well as the case without information sharing. Hence, when we pay attention to the shared information and adjust the inventory level accordingly, the supply chain can do no worse with information sharing than without. This leads to the inequality $ET^s \geq ET$, meaning that the performance of the entire supply chain improves.

Next, under the supply chain contract proposed in Section 4.3, the supply chain profit is proportionally split between the retailer and manufacturer. For instance, the retailer's time-average profit is $\rho \cdot ET$ and $\rho \cdot ET^s$ for the baseline and information-sharing model, respectively. Hence, both the retailer and manufacturer are better off with information sharing as $ET^s \geq ET$. ■

H. Proof of Corollary 2

Suppose that the manufacturer misleads the retailer about j, then the retailer will set the base-stock level to be y_{il}^s $(l = 1 - j)$, which is not optimal for $T^s(i, x, j)$. Note that the manufacturer's optimal profit by revealing the truth is $(1 - \rho) \cdot T^s(i, x, j)$. Hence, the manufacturer is worse off by misleading the retailer. ■

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