Online Supplement for "Multiperiod Models with Capacities in Competitive Supply Chain" by Georgia Perakis and Marina Zaretsky

A Extensions: expandable capacities and nonlinear costs

A.1 Expandable capacities

Consider a model in which during period t each supplier starts with an initial capacity $K_{i,0}^t$ and has an option of expanding his/her capacity up to $K_{i,1}^t$ at a cost ζ_i^t. The total cost of producing q_i^t units of product is $s_i^t(q) = s_i q + \zeta_i^t (q - K_{i,0}^t)^+$, where $0 \leq q \leq K_{i,1}^t$. Suppose also that each supplier i announces to the retailer a price schedule $w_i^t(q) = w_i^t q + v_i^t (q - K_{i,0}^t)^+$, where $0 \leq q \leq K_{i,1}^t$. In this setup, we can show that the equilibrium solution is equivalent to that of a problem with fixed capacities, that is defined as follows: the model consists of a set of $2n$ suppliers, with costs $\tilde{s}_i^t$, s.t. $\tilde{s}_i^t = s_i,\ \tilde{s}_{i+n}^t = s_i + \zeta_i^t$ and capacities $\tilde{K}_i^t = K_{i,0}, \tilde{K}_{i+n} = K_{i,1} - K_{i,0}$, for $1 \leq i \leq n$. Let us denote the suppliers' prices and quantities in this new model as $\tilde{w}_i$s and $\tilde{q}_i$s.

In order to see that the problems are equivalent, we first observe that the retailer's problem can be formulated as

$$\max_{\mathbf{q}^t, \mathbf{y}^t \geq 0} \left\{ \sum_{t=1}^{T} \Big(r(D^t - I^t + q^t + y^t) - \mathbf{w}^{t\prime}\mathbf{q}^t - \mathbf{v}^{t\prime}\mathbf{y}^t - hI^{t+} - bI^{t-} \Big) \right. \\ \left. \mid \mathbf{q}^t \leq \mathbf{K}_0^t,\ \mathbf{y}^t \leq \mathbf{K}_1^t - \mathbf{K}_0^t, I^{t+1} = (I^t + q^t + y^t - D^t)^+, I^1 = I_0 \right\}. \quad (21)$$

Therefore, the retailer's problem is equivalent to a problem with $2n$ suppliers whose prices and capacities are $\mathbf{w}, \mathbf{K}_0$ and $\mathbf{v}, \mathbf{K}_1 - \mathbf{K}_0$. Supplier i's problem can also be formulated as follows:

$$\max_{w_i^t, v_i^t \geq 0} \sum_{t=1}^{T} \big((w_i^t - s_i) q_i^t(\mathbf{w}^t, \mathbf{v}^t) + (v_i^t - s_i - \zeta_i^t) y_i^t(\mathbf{w}^t, \mathbf{v}^t) \big). \quad (22)$$

Here q_i^t is the amount that is ordered from the first $K_{i,0}^t$ units of capacity and y_i^t is the amount that is ordered from the last $K_{i,1}^t - K_{i,0}^t$ units, left in supplier i's capacity. During each period this problem is separable in terms of variables v_i and w_i. Therefore, each supplier with a variable capacity can be modeled as two suppliers with fixed capacities $K_{i,0}$ and $K_{i,1} - K_{i,0}$ and costs s_i and $s_i + \zeta_i^t$.

So far we have assumed that extra capacity is available at each period. Nevertheless, using it does not affect the initial level of capacity that is available in the next period. This is due to the fact that when the next period capacity is set as

$$K_{i,0}^{t+1} = K_{i,0}^t + y_i^t, \tag{23}$$

supplier i's problem remains separable. On the other hand, with this capacity equations, the retailer's problem has an additional constraint on capacities, namely, $q_i^t \leq K^{t-1} + y^{t-1}$. Since this type of constraint is linear, the properties of the solution to the retailer's problem remain the same. Therefore, the model with expandable capacities (i.e., where (23) holds) can still be reformulated as a $2n$-supplier/single-retailer decentralized supply chain competition problem.

Finally, suppose that there is some additional constant charge θ_i^t that supplier i incurs whenever he/she makes the decision to expand capacity (i.e., supplier i's cost is $s_i^t(q) = s_i q + \zeta_i^t (q - K_{i,0}^t)^+ + 1_{q > K_{i,0}^t} \theta_i^t$, where $0 \leq q \leq K_{i,1}^t$). In this case, supplier i can again be split into two suppliers: one that seeks to maximize $(w - s_i) q_i(\mathbf{w}, \mathbf{v})$ and another supplier, maximizing $(v - \zeta_i^t - s_i) y_i(\mathbf{w}, \mathbf{v}) - \theta_i^t$. Solving these two maximization problems will allow supplier i to set his/her equilibrium price v high enough so the capacity expansion is profitable.

A.2 Piecewise linear and nonlinear costs

The following theorem generalizes the discussion above.

Theorem 5 *The model in which suppliers have bounded piecewise linear increasing costs is equivalent to a model with fixed capacities.*

The case of piecewise linear increasing costs can be generalized to the case where costs are piecewise convex increasing functions such that their slopes are increasing in the intervals where the functions are constant. When these types of costs are used and the retailer's revenue is a convex function. Then function $q(\mathbf{w}(\cdot))$ is well defined (through the retailer's problem) and, therefore, the supplier problem are well defined and an equilibrium exists (nevertheless, we need additional conditions for the existence of a pure strategy equilibrium).

Finally, we note that the convex costs assumption might not hold. For example, given that there are economies of scale, the costs should generally decrease as the quantities rise. However, when a supplier needs to install new capacities/technologies, the costs might be convex at least for a short period of time, and our analysis would provide a framework for studying such situations.