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Appendix for "Sourcing through Auctions and Audits"

This appendix provides the technical proofs for the paper "Sourcing through Auctions and Audits."

Proof of Proposition 3

Suppose that the buyer publishes her true benefit function $R(\mathbf{x})$. Given that supplier 1 wins the auction, her payoff after the auction is $\max_{\mathbf{x}}\{R(\mathbf{x})-C_1(\mathbf{x})-b_2\}$. Since the highest losing bid b_2 does not affect supplier 1's payoff, it is in her best interest to select the contract level $\mathbf{x}$ that maximizes the supply chain profit. Therefore, the value of winning such a contract for supplier 1 is Π_1, and likewise for other suppliers when they win the auction. Also, given that the auction is of the "second price" type, it follows that it is a dominant strategy for each buyer to bid exactly her value, i.e., $b_i = \Pi_i$ for every supplier. Thus, the winning supplier's profit is $\Pi_1 - \Pi_2$ and the buyer's profit is Π_2. Q.E.D.

Proof of Proposition 4

After winning the auction, supplier 1's goal is to find $\mathbf{x}$ and b that solve the following optimization problem:

$$\max_{\mathbf{x},b}\{b - C_1(\mathbf{x})|R(\mathbf{x}) - b = S(\mathbf{x}_2, b_2)\},$$

where b is the payment supplier 1 receives from the buyer and $C_1(\mathbf{x})$ is her private cost. The above optimization problem can be simplified as $\max_{\mathbf{x}}\{R(\mathbf{x}) - C_1(\mathbf{x}) - S(\mathbf{x}_2, b_2)\}$, where we have replaced the price b by $R(\mathbf{x}) - S(\mathbf{x}_2, b_2)$. Since $S(\mathbf{x}_2, b_2)$ does not affect the choice of $\mathbf{x}$, supplier 1 will choose the contract that maximizes the supply chain profit, i.e., $\mathbf{x}_1^*$. Following this, her payoff after the auction is $R(\mathbf{x}_1^*) - C_1(\mathbf{x}_1^*) - S(\mathbf{x}_2, b_2)$.

We can now determine the suppliers' equilibrium bidding strategies. Since both the probability of winning the auction and the payoff after the auction are unaffected by her own bid, supplier 1's dominant strategy is to bid such that she breaks even if the highest losing score equals her true valuation of the supply contract. In other words, $S(\mathbf{x}_1, b_1) = R(\mathbf{x}_1^*) - C_1(\mathbf{x}_1^*)$. This can be implemented if she chooses $\mathbf{x}_1 = \mathbf{x}_1^*$ and $b_1 = C_1(\mathbf{x}_1^*)$. Similarly, a dominant strategy for each supplier i is to report $S(\mathbf{x}_i, b_i) = R(\mathbf{x}_i^*) - C_i(\mathbf{x}_i^*)$. In equilibrium, supplier 1's net payoff is $R(\mathbf{x}_1^*) - C_1(\mathbf{x}_1^*) - S(\mathbf{x}_2, b_2) = \Pi_1 - \Pi_2$. This implies that the buyer's payoff is Π_2. Q.E.D.

Proof of Proposition 5

We use the theory of auctions with *contingent payments* introduced in Hansen (1985) to establish the proposition. Since the payment by the winning supplier is independent of the winning supplier's bid, it is a dominant strategy for each supplier to bid exactly her value, i.e., $b_i = \Pi_i$ for every supplier. Also, once supplier 1 wins the contract, her payoff is

$$-\mu b_2 - (1-\mu)\mathbb{E}\left[R(\mathbf{x}) - \widetilde{C}_1(\mathbf{x})\right] + R(\mathbf{x}) - C_1(\mathbf{x}) = \mu\left[R(\mathbf{x}) - C_1(\mathbf{x})\right] - \mu b_2,$$

since the expected value of $\widetilde{C}_1(\mathbf{x})$ equals $C_1(\mathbf{x})$. Therefore, it is in supplier 1's best interest to select $\mathbf{x}$ that maximizes the supply chain profit. Because supplier 1's bid does not affect her payoff after the auction, bidding Π_1 is her dominant strategy (likewise for other suppliers). Thus, the winning bidder's profit is $\mu\left[\Pi_1 - \Pi_2\right]$ and the buyer's profit is $\mu\Pi_2 + (1-\mu)\Pi_1$. By varying μ in the interval $(0, 1]$, the buyer's profits can cover the entire interval $[\Pi_2, \Pi_1)$. Q.E.D.

Proof of Proposition 6

We drop the dependence on μ for ease of notation. Suppose supplier i wins the auction and supplier j is the highest loser. Her expected payoff is given by:

$$-\mu b_j - (1-\mu)[R(\mathbf{x}) - D_i(\mathbf{x}, e)] + R(\mathbf{x}) - C_i(\mathbf{x}, e) = \mu[(R(\mathbf{x}) - C_i(\mathbf{x}, e) - \frac{1-\mu}{\mu}E_i(\mathbf{x}, e)) - b_j].$$

Since b_j does not affect supplier i's choice of the contract and effort, she should choose $\mathbf{x}_i^\mu$ and e_i^μ after winning the auction. Moreover, her bid does not affect the payment and the profit-sharing. This implies that supplier i's dominant strategy is to bid

$$b_i = d_i = \max_{\mathbf{x}, e}\left\{R(\mathbf{x}) - C_i(\mathbf{x}, e) - \frac{1-\mu}{\mu}E_i(\mathbf{x}, e)\right\}.$$

Thus, the supplier with the highest d_i (i.e., $d_{(1)}$) wins the auction. The buyer's expected profit is equal to

$$\begin{aligned}
& \mu d_{(2)} + (1-\mu)\left[R(\mathbf{x}^\mu_{(1)}) - D_{(1)}(\mathbf{x}^\mu_{(1)}, e^\mu_{(1)})\right] \\
= \quad & \mu d_{(2)} + (1-\mu)\left[R(\mathbf{x}^\mu_{(1)}) - C_{(1)}(\mathbf{x}^\mu_{(1)}, e^\mu_{(1)})\right] + (1-\mu)E_{(1)}(\mathbf{x}^\mu_{(1)}, e^\mu_{(1)}) \\
= \quad & \mu d_{(2)} + (1-\mu)d_{(1)} + \frac{1-\mu}{\mu}E_{(1)}(\mathbf{x}^\mu_{(1)}, e^\mu_{(1)}).
\end{aligned}$$

Q.E.D.

Proof of Proposition 7

¿From the proof to Proposition 6, $d_i = \max_{\mathbf{x},e} \left\{ R(\mathbf{x}) - C_i(\mathbf{x}, e) - \frac{1-\mu}{\mu} E_i(\mathbf{x}, e) \right\}$. Due to the fact that $(1-\mu)/\mu$ is a decreasing function of μ, for every fixed $\mathbf{x}$ and e and $\mu_1 < \mu_2$,

$$R(\mathbf{x}) - C_i(\mathbf{x}, e) - \frac{1-\mu_2}{\mu_2} E_i(\mathbf{x}, e) \geq R(\mathbf{x}) - C_i(\mathbf{x}, e) - \frac{1-\mu_1}{\mu_1} E_i(\mathbf{x}, e).$$

Thus, the maximum of the left hand side of the inequality (over $\mathbf{x}$ and e) is not smaller than the maximum of the right hand side. This proves (a). The proof of (b) follows from observing that when $\mu = 1$ and the definition of $d_i(\mu)$, the supplier maximizes the channel profit, $R(\mathbf{x}) - C_i(\mathbf{x}, e)$. Q.E.D.

Proof of Lemma 1

Recall that $\mathbf{x}_i^\mu$ and e_i^μ are the optimal contract and effort level for supplier i. Then,

$$\begin{aligned}
& \frac{d}{d\mu}[R(\mathbf{x}_i^\mu) - C_i(\mathbf{x}_i^\mu, e_i^\mu)] \\
= \; & \frac{d}{d\mu}\left(R(\mathbf{x}_i^\mu) - C_i(\mathbf{x}_i^\mu, e_i^\mu) - \frac{1-\mu}{\mu} E_i(\mathbf{x}_i^\mu, e_i^\mu) \right) + \frac{d}{d\mu}\left(\frac{1-\mu}{\mu} E_i(\mathbf{x}_i^\mu, e_i^\mu) \right) \\
= \; & -\frac{d}{d\mu}\left(\frac{1-\mu}{\mu} \right) E_i(\mathbf{x}_i^\mu, e_i^\mu) + \frac{d}{d\mu}\left(\frac{1-\mu}{\mu} E_i(\mathbf{x}_i^\mu, e_i^\mu) \right) \\
= \; & \left(\frac{1-\mu}{\mu} \right) \frac{d}{d\mu} E_i(\mathbf{x}_i^\mu, e_i^\mu) \geq 0,
\end{aligned}$$

where the second equality follows from the envelope theorem and the final inequality is due to the assumption of the proposition. Q.E.D.

Proof of Proposition 8

Suppose that a type-θ supplier wins the auction and the highest loser is type-κ. The winning supplier's expected payoff under the AAU mechanism is:

$$\begin{aligned}
& -\mu\beta^U(\kappa) - (1-\mu)\left[R(\mathbf{x}) - D(\mathbf{x}, e, \theta) - (R(\mathbf{x}) - D(\mathbf{x}, e, \theta) - \beta^U(\theta)) \right] \\
& + R(\mathbf{x}) - D(\mathbf{x}, e, \theta) - E(\mathbf{x}, e, \theta) \\
= \; & -\mu\beta^U(\kappa) - (1-\mu)\beta^U(\theta) + R(\mathbf{x}) - D(\mathbf{x}, e, \theta) - E(\mathbf{x}, e, \theta).
\end{aligned}$$

Since the first two terms do not affect the winning supplier's choice of contract and effort, she will choose those that maximize $R(\mathbf{x}) - D(\mathbf{x}, e, \theta) - E(\mathbf{x}, e, \theta)$, i.e., $(\mathbf{x}^*(\theta), e^*(\theta))$. Moreover, since the optimal contract and effort level $(\mathbf{x}, e)$ do not depend on her bid, even if a type-θ

supplier pretends to be type-τ, she still chooses $(\mathbf{x}, e) = (\mathbf{x}^*(\theta), e^*(\theta))$ based on her true type.

Now we characterize the equilibrium bidding function. Let us first assume that in a symmetric equilibrium, the bidding function is strictly decreasing. Under this assumption, the supplier with the lowest cost wins the auction, and therefore supply chain efficiency is achieved. Given this monotonicity, the probability that all other $n-1$ suppliers' parameters are higher than a fixed number s with probability $(1-F(s))^{n-1}$, the bid b_2 equals to $\beta(s)$ with probability density $(n-1)(1-F(s))^{n-2}f(s)$. We now write down the winning supplier's expected payoff if her true type is θ but she pretends to be type-τ:

$$\Pi^U(\tau|\theta) \equiv \int_\tau^\infty \{-\mu\beta^U(s) - (1-\mu)\beta^U(\tau) + V(\theta)\}(n-1)(1-F(s))^{n-2}f(s)ds.$$

A necessary condition for the supplier to bid truthfully is that the above expected payoff is maximized when $\tau = \theta$. Therefore, we can differentiate it by τ and obtain the derivative of her expected payoff:

$$\begin{aligned}\frac{\partial \Pi^U(\tau|\theta)}{\partial \tau} &= -(n-1)(1-F(\tau))^{n-2}f(\tau)\{-\mu\beta^U(\tau) - (1-\mu)\beta^U(\tau) + V(\theta)\} \\ &\quad +(1-F(\tau))^{n-1}[-(1-\mu)(\beta^U)'(\tau)].\end{aligned}$$

Let $\tau = \theta$ and make the above derivative zero. The first-order condition leads to a candidate bidding function that satisfies

$$(n-1)f(\theta)\beta^U(\theta) - (1-F(\theta))(1-\mu)(\beta^U)'(\theta) - (n-1)f(\theta)V(\theta) = 0,$$

which can be obtained from the above ordinary differential equation.

The boundary condition $\beta^U(\theta) = 0, \forall\theta \geq \bar{\theta}$ can be rationalized as follows. First of all, no negative bid can be sustained in equilibrium since the buyer then would incur a loss. Suppose that $\beta^U(\theta^1) > 0$ for some $\theta^1 \geq \bar{\theta}$. If there is no other type that bids below $\beta^U(\theta^1)$, then reducing the bid $\beta^U(\theta^1)$ to 0 does not hurt the type-θ^1 supplier. Therefore, it suffices to consider the case when some other types bid below $\beta^U(\theta^1)$. Since $\beta^U(\theta)$ is continuous, with strictly positive probability the type-θ^1 supplier wins the auction. She then pays a weighted sum of her own bid and the highest losing bid (both being nonnegative). However, her gross profit from the supply contract is 0, and therefore the supplier incurs a loss by winning the auction. This leads to a contradiction.

We now show that this bidding function indeed sustains an equilibrium. To this end, we need to verify that a type-θ supplier's payoff attains its maximum when she bids truthfully and that the bidding function is indeed strictly monotonic.

Plugging in the bidding function, we obtain that

$$\begin{aligned}
\frac{\partial \Pi^U(\tau|\theta)}{\partial \tau} &= -(n-1)(1-F(\tau))^{n-2} f(\tau)\{-\beta^U(\tau)+V(\theta)\} + (1-F(\tau))^{n-1}[-(1-\mu)(\beta^U)'(\tau)] \\
&= (1-F(\tau))^{n-2}\left\{(n-1)f(\tau)\beta^U(\tau) - (1-F(\tau))(1-\mu)(\beta^U)'(\tau)\right\} \\
&\quad -(n-1)(1-F(\tau))^{n-2} f(\tau)V(\theta) \\
&= (n-1)(1-F(\tau))^{n-2} f(\tau)(V(\tau)-V(\theta)),
\end{aligned}$$

where the last equality follows from the ordinary differential equation (4). According to Assumption (9), if $\tau < \theta$, $V(\tau) > V(\theta)$, and therefore $\frac{\partial \Pi^U(\tau|\theta)}{\partial \tau} > 0$. This implies that the supplier is better off by reporting a higher type. Likewise, when $\tau > \theta$, the supplier wants to report a lower type. Her profit hence attains its maximum when she reports her type truthfully. This gives us the sufficiency.

Finally, consider the monotonicity of $\beta^U(\theta)$. Suppose $\beta^U(\theta^0) > V(\theta^0)$ for some θ^0 in the interior. Since

$$(\beta^U)'(\theta) = \frac{(n-1)f(\theta)}{(1-\mu)(1-F(\theta))}\left\{\beta^U(\theta) - V(\theta)\right\},$$

$(\beta^U)'(\theta) > 0$ at $\theta = \theta^0$. Moreover, $V(\theta)$ is decreasing in θ by Assumption (9), and therefore $\beta^U(\theta) > V(\theta), \forall \theta \geq \theta^0$. However, the boundary condition shows that $\beta^U(\theta) = 0, \forall \theta \geq \bar{\theta}$, a contradiction. This implies that $\beta^U(\theta) \leq V(\theta), \forall \theta$. The strict monotonicity for all $\theta < \bar{\theta}$ follows from the ordinary differential equation. Q.E.D.

Proof of Proposition 9

Consider a supplier's payoff when she wins the auction. Her payoff depends on the revenue she gets from the supply contract and the payment she makes to the buyer in the auction. Since the same contract and effort are chosen after the auction, the gross revenue (from the supply contract) is identical under the two mechanisms. This in fact corresponds to the supplier's gross valuation of the supply contract. In the auction stage, the supply contract can be regarded as the auction object. Therefore, according to Chen (2001), this is precisely a single-object auction. Note that the "highest" type here brings the lowest supply chain profit. The revenue equivalence theorem applies directly given the conditions stated in the proposition. Q.E.D.

Proof of Proposition 10

Let us define $\mathbf{x}^K(\theta)$ and $e^K(\theta)$ as the contract and the effort level selected by the winning supplier if her type is θ, respectively. Suppose that a type-θ supplier wins the auction. Her expected payoff after winning the auction is

$$\begin{aligned} & -\mu b_2 + \mu \left[R(\mathbf{x}^K(\theta))) - D(\mathbf{x}^K(\theta), e^K(\theta), \theta)\right] \\ & +(1-\mu)E(\mathbf{x}^K(\theta), e^K(\theta), \theta) - E(\mathbf{x}^K(\theta), e^K(\theta), \theta) \\ = & -\mu b_2 + \mu \left[R(\mathbf{x}^K(\theta)) - D(\mathbf{x}^K(\theta), e^K(\theta), \theta) - E(\mathbf{x}^K(\theta), e^K(\theta), \theta)\right]. \end{aligned} \tag{8}$$

After removing redundant terms, we obtain that $(\mathbf{x}^K(\theta), e^K(\theta)) = \arg\max\{R(\mathbf{x}) - D(\mathbf{x}, e, \theta) - E(\mathbf{x}, e, \theta)\} = (\mathbf{x}^*(\theta), e^*(\theta))$, i.e., they maximize the supply chain profit.

We now derive the equilibrium bidding function. To this end, we first assume that there exists a symmetric Bayesian Nash equilibrium, in which each supplier adopts the same bidding strategy $\beta^K(\theta)$ that is strictly decreasing in θ. Based on this, we characterize a candidate bidding function. We then verify that this candidate bidding function indeed leads to an equilibrium. Since the bidding function is decreasing, the buyer can infer the supplier's type after receiving the bid. Therefore, if a type-θ supplier bids truthfully and wins the auction, the buyer expects that the unobservable part is $\hat{E}(b_1, D(x, e, \theta)) = E(\mathbf{x}^*(\theta), e^*(\theta), \theta)$.

Given the AAK mechanism, when a type-θ supplier bids as if she is type-τ, her expected payoff is

$$\begin{aligned} & -\mu b_2 - (1-\mu)\left[R(\mathbf{x}(\tau)) - D(\mathbf{x}(\tau), e(\tau), \tau) - E(\mathbf{x}(\tau), e(\tau), \tau)\right] \\ & +R(\mathbf{x}(\theta, \tau)) - D(\mathbf{x}(\theta, \tau), e(\theta, \tau), \theta) - E(\mathbf{x}(\theta, \tau), e(\theta, \tau), \theta), \end{aligned}$$

where b_2 is the second-highest bid, $R(\mathbf{x}(\tau)) - D(\mathbf{x}(\tau), e(\tau), \tau) - E(\mathbf{x}(\tau), e(\tau, \tau), \tau) = V(\tau)$ is the estimated supply chain profit, and $R(\mathbf{x}(\theta, \tau)) - D(\mathbf{x}(\theta, \tau), e(\theta, \tau), \theta) - E(\mathbf{x}(\tau), e(\theta, \tau), \theta)$ is the actual profit of the supplier. It remains to consider whether the supplier is willing to bid truthfully. Rewriting the supplier's expected payoff after she wins the auction as

$$-\mu b_2 + \mu\left[R(\mathbf{x}(\tau)) - D(\mathbf{x}(\tau), e(\tau), \tau)\right] + (1-\mu)E(\mathbf{x}(\tau), e(\tau, \tau), \tau) - E(\mathbf{x}(\theta, \tau), e(\theta, \tau), \theta),$$

we can write down the winning supplier's expected payoff if her true type is θ but she pretends to be type-τ:

$$\begin{aligned} \Pi^K(\tau|\theta) = & \int_\tau^\infty \{-\mu\beta^K(s) + \mu\left[R(\mathbf{x}(\tau)) - D(\mathbf{x}(\tau), e(\tau), \tau)\right] \\ & +(1-\mu)E(\mathbf{x}(\tau), e(\tau, \tau), \tau) - E(\mathbf{x}(\theta, \tau), e(\theta, \tau), \theta)\}(n-1)(1-F(s))^{n-2}f(s)ds, \end{aligned}$$

where $(n-1)(1-F(s))^{n-2}f(s)$ is the probability density that b_2 equals $\beta^K(s)$.

A necessary condition for the supplier to bid truthfully is that the above expected payoff is maximized when $\tau = \theta$. Therefore, we first differentiate $\Pi^K(\tau|\theta)$ by τ:

$$\begin{aligned}
\frac{\partial \Pi^K(\tau|\theta)}{\partial \tau} &= (n-1)(1-F(\tau))^{n-2}f(\tau)\{-\mu\beta^K(\tau) + \mu\,[R(\mathbf{x}(\tau)) - D(\mathbf{x}(\tau), e(\tau), \tau)] \\
&\quad +(1-\mu)E(\mathbf{x}(\tau), e(\tau,\tau), \tau) - E(\mathbf{x}(\theta,\tau), e(\theta,\tau), \theta)\} \\
&\quad +(1-F(\tau))^{n-1}\frac{\partial}{\partial \tau}\{\mu[R(\mathbf{x}(\tau)) - D(\mathbf{x}(\tau), e(\tau), \tau)] \\
&\quad +(1-\mu)E(\mathbf{x}(\tau), e(\tau), \tau) - E(\mathbf{x}(\theta,\tau), e(\theta,\tau), \theta)\}.
\end{aligned}$$

Let $\tau = \theta$ and make the above derivative zero. It gives rise to a candidate equilibrium bidding function $\beta^K(\theta)$:

$$\begin{aligned}
\beta^K(\theta) &= V(\theta) + \frac{1-F(\theta)}{\mu(n-1)f(\theta)}\frac{\partial}{\partial \tau}\{\mu V(\tau) + E(\mathbf{x}(\tau), e(\tau), \tau) - E(\mathbf{x}(\theta,\tau), e(\theta,\tau), \theta)\}|_{\tau=\theta} \\
&= V(\theta) + \frac{1-F(\theta)}{(n-1)f(\theta)}V'(\theta) \\
&\quad + \frac{1-F(\theta)}{\mu(n-1)f(\theta)}\frac{\partial}{\partial \tau}\left\{E(\mathbf{x}(\tau), e(\tau), \tau) - E(\mathbf{x}(\theta,\tau), e(\theta,\tau), \theta)\right\}|_{\tau=\theta}.
\end{aligned}$$

The regular monotone hazard rate condition implies that $\frac{1-F(\theta)}{(n-1)f(\theta)}$ is decreasing in θ. From Assumption (10), we know that both $V'(\theta)$ and $\frac{\partial}{\partial \tau}\left\{E(\mathbf{x}(\tau), e(\tau), \tau) - E(\mathbf{x}(\theta,\tau), e(\theta,\tau), \theta)\right\}|_{\tau=\theta}$ are strictly decreasing in θ. Therefore, the bidding function $\beta^K(\theta)$ is monotonic.

To verify that this is indeed an equilibrium, we have to show that the supplier's payoff attains its maximum when she bids truthfully. Plugging in the candidate bidding function $\beta^K(\theta)$ into $\frac{\partial \Pi^K(\tau|\theta)}{\partial \tau}$, we obtain

$$
\begin{aligned}
\frac{\partial \Pi^K(\tau|\theta)}{\partial \tau} &= -(n-1)(1-F(\tau))^{n-2} f(\tau)\mu\beta^K(\tau) \\
&\quad +(n-1)(1-F(\tau))^{n-2} f(\tau)\{\mu\,[R(\mathbf{x}(\tau)) - D(\mathbf{x}(\tau), e(\tau), \tau)] \\
&\quad +(1-\mu)E(\mathbf{x}(\tau), e(\tau), \tau) - E(\mathbf{x}(\theta,\tau), e(\theta,\tau), \theta)\} \\
&\quad +(1-F(\tau))^{n-1}\frac{\partial}{\partial \tau}\{\mu[R(\mathbf{x}(\tau)) - D(\mathbf{x}(\tau), e(\tau), \tau)] \\
&\quad +(1-\mu)E(\mathbf{x}(\tau), e(\tau), \tau) - E(\mathbf{x}(\theta,\tau), e(\theta,\tau), \theta)\} \\
&= -(n-1)(1-F(\tau))^{n-2} f(\tau) \times \\
&\quad \left\{\mu[V(\tau) + \frac{1-F(\tau)}{\mu(n-1)f(\tau)}\frac{\partial}{\partial z}\{\mu V(z) + E(\mathbf{x}(z), e(z), z) - E(\mathbf{x}(\tau,z), e(\tau,z), \tau)\}|_{z=\tau}]\right\} \\
&\quad +(n-1)(1-F(\tau))^{n-2} f(\tau)\{\mu V(\tau) + E(\mathbf{x}(\tau), e(\tau), \tau) - E(\mathbf{x}(\theta,\tau), e(\theta,\tau), \theta)\} \\
&\quad +(1-F(\tau))^{n-1}\frac{\partial}{\partial z}\{\mu V(z) + E(\mathbf{x}(\mathbf{z}), e(z), z) - E(\mathbf{x}(\theta,z), e(\theta,z), \theta)\}|_{z=\tau} \\
&= (n-1)(1-F(\tau))^{n-2} f(\tau)\{E(\mathbf{x}(\tau), e(\tau), \tau) - E(\mathbf{x}(\theta,\tau), e(\theta,\tau), \theta)\} \\
&\quad +(1-F(\tau))^{n-1}\frac{\partial}{\partial z}\{E(\mathbf{x}(\tau,z), e(\tau,z), \tau) - E(\mathbf{x}(\theta,z), e(\theta,z), \theta)\}|_{z=\tau}.
\end{aligned}
$$

When $\tau < \theta$, $E(\mathbf{x}(\tau), e(\tau), \tau) - E(\mathbf{x}(\theta,\tau), e(\theta,\tau), \theta) > 0$ and $\frac{\partial}{\partial z}E(\mathbf{x}(\theta,z), e(\theta,z), \theta)$ is increasing in θ by Assumption (10). Therefore, $\frac{\partial \Pi^K(\tau|\theta)}{\partial \tau} < 0$ whenever $\tau < \theta$. This implies that the supplier intends to report a higher type, or equivalently, submit a lower bid. Similarly, if $\tau > \theta$, the supplier finds it profitable to increase her bid. This shows that $\Pi^K(\tau|\theta)$ is unimodal in τ and attains its maximum at $\tau = \theta$. Q.E.D.

Proof of Corollary 1

¿From Equation (8), if we let μ approach 0, the expected payoff of the winning supplier vanishes. Hence the corollary is true. Q.E.D.

Proof of Proposition 11

Recall that in the AAK mechanism, supply chain efficiency is achieved and the buyer can allocate arbitrarily the profit by changing μ. Therefore, the AAK mechanism performs better than the DSC mechanism (which results in inefficient contract and effort and leaves the winning supplier information rent). Q.E.D.